Linear Discriminant Analysis on Brain Computer Interface.

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Abstract **– This report analyses the application of Linear Discriminant Analysis in Brain Computer Interface technology. It is aimed to obtain an objective evaluation of the discrimination capability achieved when different filtering windows are considered in order to differentiate between three different cerebral activities.**

In this report the following issues are discussed:

- **Quantification of the discrimination capability between the employed cerebral activities.**
- **Identification of the frequency bands with the highest discrimination power.**
- **Methodology to weight the amplitude of the previous frequency bands in order to reduce the dimensionality of the feature space and facilitate posterior analysis, without lost of intrinsic characteristics of each cerebral activity.**
- **Determination of the best preprocess window.**

Linear Discrimination Analysis is employed in order to reduce the dimensionality of the input feature space; bilateral contrasts between features, inferred from each cerebral activity, are used to determine the discrimination power.

Keywords **– Brain Computer Interface; Electroencephalography; Linear Discriminant Analysis; Pattern recognition; Spectral Analysis.**

I. INTRODUCTION.

Brain Computer Interface technology, BCI, is aimed to communicate human beings with external computerized devices using the encephalographic signal as primary source of commands [1][2][3][4]; in the first international meeting for BCI technology in 1999 it was established that BCI *"must not depend on the brain's normal output pathways of peripheral nerves and muscles"*.

A variety of methods for monitoring brain activity might serve in BCI technology: electroencephalography (EEG) and more invasive electrophysiological methods, magnetoencephalography (MEG), positron emission tomography (PET), functional magnetic resonance imaging (fMRI), and optical imaging. However, MEG, PET, fMRI, and optical imaging are still technically demanding, some of them depend on blood flow and have long time constants and thus are less amenable to fast communication. At present, only EEG meets the requirements of short time constant, affordable cost, and it is relatively simple to implement. Different approaches are considered in current BCI research, they include standard scalp-recorded EEG as well as those that use epidural, subdural, or intracortical recording. While all these BCIs use electrophysiological methods, the basic principles of BCI design and operation apply also to BCIs that use other methods to monitor brain activity.

In order to control an external device using thoughts it is necessary to associate some mental patterns to device commands, so an algorithm that detects, acquires, filters and classifies the human electroencephalographic signal is required [2][5][6][7]. Usually all BCI systems are compounded from the following parts:

- *Signal acquisition*. Electrophysiological BCIs can be categorized by whether they use non-invasive or invasive methodology. They can also be categorized by whether they use evoked or spontaneous inputs. In the signal-acquisition part of BCI operation, the chosen input is acquired by the recording electrodes, amplified, and digitized.
- *Signal processing: feature extraction.* The digitized signal are then subjected to one or more of a variety of feature extraction procedures, such as spatial filtering, voltage amplitude measurements, spectral analysis, or single-neuron separation. This analysis extracts the signal features that encode the user's messages or commands. BCIs can use signal features that are in the time domain (e.g. evoked potential amplitudes or neuronal firing rates) or the frequency domain (e.g. mu or beta-rhythm amplitudes)

[8]. A BCI could conceivably use both time-domain and frequency-domain signal features, and might thereby improve performance. It is also possible for a BCI to use signal features, like sets of autoregressive parameters, that correlate with the user's intent but not necessarily reflect specific brain events, in those cases it is necessary to ensure that the chosen features are not contaminated by EMG, EOG or other non-CNS artifacts.

- *Signal processing: the translation algorithm.* It translates the signal features into device commands-orders that carry out the user's intent. This algorithm might use linear methods (e.g. classical statistical analysis) or nonlinear methods (e.g. neural networks). The algorithm changes independent variables (i.e. signal features) into dependent variables (i.e. device control commands).
- *The output device.* At the moment the output device is a computer screen and the output is the selection of targets, letters, or icons presented on it. Initial studies are also exploring BCI control of a neuroprothesis or orthesis that provides hand closure to people with cervical spinal cord injuries.
- *The operating protocol.* It is the protocol that guides the operation of the BCI device. It defines how the system is turned on and off, whether communication is continuous or discontinuous, or if the message transmission is triggered by the system or by the user, the sequence and speed of interactions between user and system, and what feedback is provided to the user.

BCI devices fall into two classes: dependent and independent [9]. A dependent BCI does not use the brain's normal output pathways to carry the message, but activity in these pathways is needed to generate the brain activity that does carry it. A dependent BCI is an alternative method for detecting messages carried in the brain's normal output pathways (e.g. gaze direction is detected by monitoring EEG rather than by monitoring eye position directly). An independent BCI does not depend in any way on the brain's normal output pathways. The message is not carried by peripheral nerves and muscles (e.g. P300 evoked potential).

This report focus on the applicability of LDA to BCI and how the windowing effect affects the discrimination capability of the brain proposed activities.

In the experiments considered for this report a low number of scalp-electrodes has been used to capture the endogenous electroencephalographic subject's signal. In order to facilitate the use of this technology it is important to make it easy to use, cosmesis is often crucial; that is, how the system looks and how the user looks while employing it, the number of electrodes employed in these devices is a global key feature, as the fewer of electrodes used, the higher the comfort [2][4].

Because the main changes in brain activity are associated to changes in the power amplitude of band frequencies [4], spectrograms based on FFT are used to obtain initial feature vectors. LDA technique is used to combine these initial features in order to reduce the dimensionality of the input space [10]. To minimize the leakage effect seven different types of preprocess windows has been considered: rectangular, triangular, Blackman's, Hamming's, Hanning's, Kaiser's and Tukey's [11][12][13]. The evidence of statistical difference in the feature populations associated to different brain activities has been previously shown [14].

To determine the discrimination power between the proposed cerebral activities and the effect of filtering window, a statistical procedure of bilateral contrast of independent populations has been used [15], the results of each contrast is both qualitative and quantitative, qualitative in order to accept or reject the null hypothesis of equality in the population of features, quantitative in order to compare the discrimination power through significance contrast level $\alpha = 1 - p$.

This article is composed of the following sections:
Section II briefly describes the methodology.
Section III describes the LDA technique.
Section IV explains the statistical bilateral contrasts.
Section V and VI presents and analyzes the results.
Section VII is devoted to conclusions.

II. Experimental procedure.

The tests described below were carried out on five male healthy subjects, one of them has been trained before, but the other four were novice in the use of the system.

In order to facilitate the mental concentration on the proposed activities, the experiments were performed in a room with low level of noise and under controlled environmental conditions, all electronic equipments external to the experiment around subject were switched off to avoid electromagnetic artifacts. The subjects were sat down in front of the acquisition system monitor, at 50 cm from the screen, their hands were in a visible position, the supervisor of the experiment controlled the correct development of it [16][17].

A. Methodology.

The experimental process is shown on figure 1.

Test of system devices. Checks the correct level of battery, and the correct state of the electrodes.

System assembly. Device connections: superficial electrodes (Grass Au-Cu), battery, bio-amplifier (g.BSamp by g.tec), acquisition signal card (PCI-MIO-16/E-4 by National Instrument), computer.

System test. Verifies the correct operation of the whole system. To minimize noise from the electrical network the Notch filter (50Hz) of the bio-amplifier is switched on.

Subject preparation for the experiment. Application of electrodes on subject's head. It is verified that electrode impedance was lower than 4 KOhms.

System initialization and setup. Verification of data register.

Experiment setup. The supervisor of the experiment sets-up the number of replications, $N_{rep} = 10$, and the quantity of different mental activities, $N_{act} = 3$. The duration of each

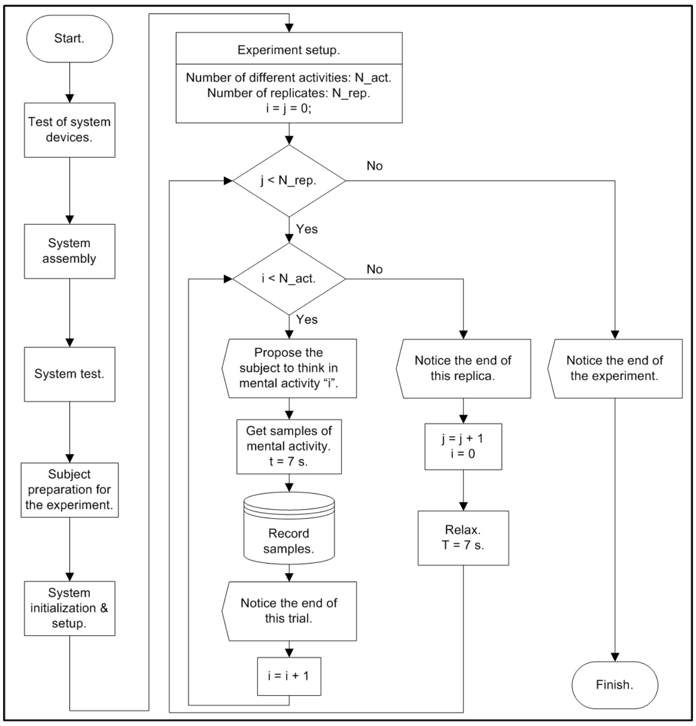

Fig. 1. DIAGRAM OF THE EXPERIMENT REALIZATION.

mental activity, a trial, is $t = 7s$, the acquisition frequency is $f_s = 384Hz$. The system randomly suggests to the subject to think about the proposed mental activity. A short relax is allowed at the end of each trial; between replications the relax time is $t = 7s$.

B. Position of the electrodes and description of cerebral activities.

Electrodes were placed in the central zone of the skull, next to C3 and C4, two pair of electrodes were placed in front of and behind of Rolandic sulcus, this zone is one with the highest discriminant power, it takes signal from motor and sensory areas of the brain [17][7]. Reference electrode was placed on the right mastoid, two more electrode are placed near to the corner of the eyes to register blinking.

Fig. 2. ELECTRODE PLACEMENT.

The supervisor of the experiment asks the subject to figure out the following mental activities, these activities will be the cerebral patterns to differentiate among them.

Activity A. Mathematical task. Recursive subtraction of a prime number, i.e. 7, from a big quantity, i.e. 3.000.000.

Activity B. Motor imagery. The subject imagines moving their limbs or hands, but without the materialization of the movement.

Activity C. Relax. The subject is relaxed.

C. Feature selection.

The registered signal is chopped in packages of samples, similar to the bundles of samples obtained from and acquisition card in an on-line BCI application. Each package has 128 samples, because each trial has $7s$ of registered signal at $f_s = 384Hz$ and no overlapping has been considered, there are 21 packages per trial. A vector of features is extracted from each package. This vector is made up as the mean of the amplitudes of the frequency bands [11]. Because the frequency of normal human brain is under 40-50Hz, only frequencies between 6 and 38Hz have been considered.

TABLE I.
FEATURE VECTOR.

FFT index.	Frequency.	Denomination.
1	0 - 2	Not considered
2	3 - 5	Not considered
3	6 - 8	θ.
4	9 - 11	α_1.
5	12 - 14	$\alpha 2$.
6 - 7	15 - 20	β_1.
8 - 10	21 - 29	β_2.
11 - 13	30 - 38	β_3.
14 - 64	39 - 192	Not considered

III. LINEAR DISCRIMINANT ANALYSIS PROCEDURE.

A. Introduction.

Linear Discriminant Analysis is a preprocess technique used in machine learning, its objective is to find the best combination of features that separate two or more types of objects or events. The result can be used as linear classifier or as a technique to reduce the feature space dimension before the classification process. LDA try to express the dependent variable as a combination of independent variables.

Supposed N classes of observations x with means $\bar{\mu_i}$ and covariances S_i. The linear combination of features $w.x$ will have means $w.\bar{\mu_y}$ and variances $w^T S_i w$ for $i = 0, N$. Fisher defined the separation between these N distributions to be the ratio of the variance between the classes to the variance within the classes.

$$T = \frac{\sigma^2_{between}}{\sigma^2_{within}} = \frac{\Sigma_{i=1}^N (w.\bar{\mu_i} - w.\bar{\mu})^2}{\Sigma_{i=1}^N w^T S_i w}$$

The maximum separation occurs when

$$T = (\Sigma_{i=1}^N S_i)^{-1} (\Sigma_{i=1}^N (\bar{\mu_i} - \bar{\mu}))$$

Under the assumptions of normally distributed classes and equal class covariances

$$T = S^{-1}.(\bar{\mu_i} - \bar{\mu})$$

B. Operational procedure.

The operational procedure followed to carry on the Linear Discriminant Analysis is described afterwards.

1. Samples from each mental tasks are obtained.
   X_a Mathematical Activity.
   $X_{b1/b2}$ Movement imagination / realization.
   X_c Relax.
2. Statistical definition of each population.
   Mean: Covariance matrix
   X_a $\bar{\mu_a}$ $Cov_a = (x_a - \bar{\mu_a})(x_a - \bar{\mu_a})^T$.
   X_b $\bar{\mu_b}$ $Cov_b = (x_b - \bar{\mu_b})(x_b - \bar{\mu_b})^T$.
   X_c $\bar{\mu_c}$ $Cov_c = (x_a - \bar{\mu_c})(x_c - \bar{\mu_c})^T$.
3. Statistical definition of all populations.
   Mean: Covariance matrix
   X_T $\bar{\mu_T}$ $Cov_T = (x - \bar{\mu_T})(x - \bar{\mu_T})^T$.
4. Within and Between covariance matrices calculation.

$$\begin{array}{ll} Within: & S_w = \Sigma_j p_j cov_j \\ Between: & S_b = \Sigma_j p_j (\bar{\mu_j} - \bar{\mu_T})(\bar{\mu_j} - \bar{\mu_T})^T \end{array}$$

   In which the probability of each population is p_j.
5. The optimizing criterion in LDA is the ratio of between-class to the within-class covariance matrices. The solution obtained by maximizing this criterion defines the axes of the transformed space.

$$C = S_w^{-1} x Sb$$

6. By definition, an eigen-vector represents a 1-D invariant subspace of vector space in which the transformation is applied. A set of these eigen-vectors with non-zero eigenvalues are linearly independent and invariant under the transformation. Thus any vector space can be represented in terms of linear combinations of the eigen-vectors.
7. Once the transformation matrices have been obtained, the data sets are transformed using LDA transform. The decision region in the transformed space is a hyperplane of lower dimension than the feature space.
   X_a $X'_a = T.X_a$.
   X_b $X'_b = T.X_b$.
   X_c $X'_c = T.X_c$.
8. Once the transformations are completed using LDA transforms, Euclidean or Mahalanobis distance can be used to classify new vectors.
9. The smallest value among the n distances classifies the new vector as belonging to class n.

IV. Statistical analysis procedure.

Bilateral contrasts between two population are used to determine if there is statistical evidence of difference between the population of features obtained from each mental activity. Each component of the vector is considered to determine its significance and separability power. Bilateral contrast makes use of population variance, if the equality of both population variances is rejected it is necessary to apply a correction factor in the degrees of freedom. These contrasts were done for each type of window.

- *Bilateral contrast to the variance ratio.*
  The equality of variances is obtained with $R = 1$.
  n_1 = sample size of the first population.
  n_2 = sample size of the second population.
  $\hat{S}_1$ = standard deviation of the first population.
  $\hat{S}_2$ = standard deviation of the second population.
  F = Fisher distribution.
  Null hypothesis H_o vs. alternative hypothesis H_1.

$$H_o : \frac{(\sigma_1)^2}{(\sigma_2)^2} = R \;\; vs. \;\; H_1 : \frac{(\sigma_1)^2}{(\sigma_2)^2} \neq R$$

  Considering that:

$$\frac{(n_1-1)\hat{S_1}^2}{\sigma_1^2} \rightsquigarrow \chi^2_{n_1-1} \qquad \frac{(n_2-1)\hat{S_2}^2}{\sigma_2^2} \rightsquigarrow \chi^2_{n_1-1}$$

$$\frac{\frac{1}{n_1-1}\frac{(n_1-1)\hat{S_1}^2}{\sigma_1^2}}{\frac{1}{n_2-1}\frac{(n_2-1)\hat{S_2}^2}{\sigma_2^2}} = \frac{\sigma_2^2}{\sigma_1^2}\frac{\hat{S_1}^2}{\hat{S_2}^2} \rightsquigarrow F_{n_1-1,n_2-1}$$

  Therefore under the fulfillment of the null hypothesis:

$$F_{Exp} = \frac{1}{R}\frac{\hat{S_1}^2}{\hat{S_2}^2} \rightsquigarrow F_{n_1-1,n_2-1}$$

  The zone of H_o acceptance is:

$$a_{teo} = F_{(n_1-1,n_2-1,1-\frac{\alpha}{2})} \;\; y \;\; b_{teo} = F_{(n_1-1,n_2-1,1-\frac{\alpha}{2})}$$

$$a_{teo} \leq F_{Exp} \leq b_{teo}$$

- *Bilateral contrast of two independent normal and homocedastic populations.* Null hypothesis H_o vs. alternative hypothesis H_1.

$$H_o : \mu_1 - \mu_2 = \Delta \;\; vs. \;\; H_1 : \mu_1 - \mu_2 \neq \Delta$$

  The variances of the both population are equal but unknown.

$$T_{Exp} = \frac{(\bar{X_1} - \bar{X_2}) - (\mu_1 - \mu_2)}{\hat{S}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

  In which $\hat{S}^2$ is the pseudo-variance of $\hat{S_1}^2$ and $\hat{S_2}^2$

$$\hat{S}^2 = \frac{(n_1-1) * \hat{S_1}^2 + (n_2-1) * \hat{S_2}^2}{n_1 + n_2 - 2}$$

  The zone of H_o acceptance is:

$$T_{Teo} = t_{(n_1+n_2-2,1-\frac{\alpha}{2})}$$

  If $|T_{Exp}| \leq T_{Teo}$ then H_o is accepted, on the contrary H_1 is accepted and H_o is rejected.
- *Bilateral contrast of two independent normal and heterocedastic populations.* The null hypothesis H_o and alternative hypothesis are similar to the previous ones, the statistical measure is:

$$T_{Exp} = \frac{(\bar{X_1} - \bar{X_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\hat{S_1}^2}{n_1} + \frac{\hat{S_2}^2}{n_2}}} \rightsquigarrow t_f$$

In which f is the number of degrees of freedom calculated with the Welch's formula:

$$f = \frac{(\frac{\hat{S_1}^2}{n_1} + \frac{\hat{S_2}^2}{n_2})^2}{\frac{1}{n_1+1}(\frac{\hat{S_1}^2}{n_1})^2 + \frac{1}{n_2+1}(\frac{\hat{S_2}^2}{n_2})^2} - 2$$

In this case the zone of H_o acceptance is:

$$T_{Teo} = t_{(f, 1-\frac{\alpha}{2})}$$

If $|T_{Exp}| \leq T_{Teo}$ then H_o is accepted, on the contrary is assumed that the populations are different.

V. Results.

The next figures show for the LDA transformed coordinates of both channels: C3'-C3" and C4'-C4", and for each type of filtering window the associated probability results p of the bilateral contrast tests between the former mental tasks. In order to represent the dispersion of the results the mode value and bars from 15th to 85th percentile have been used.

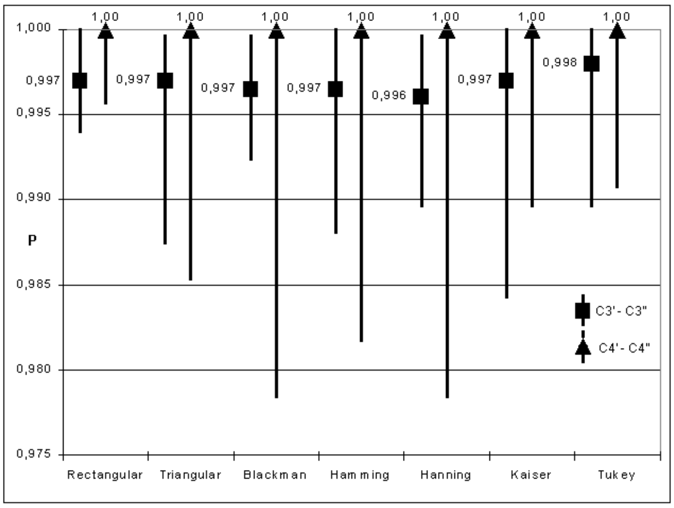

Fig. 3. Math task vs. Motor imagery. Coordinate X1.

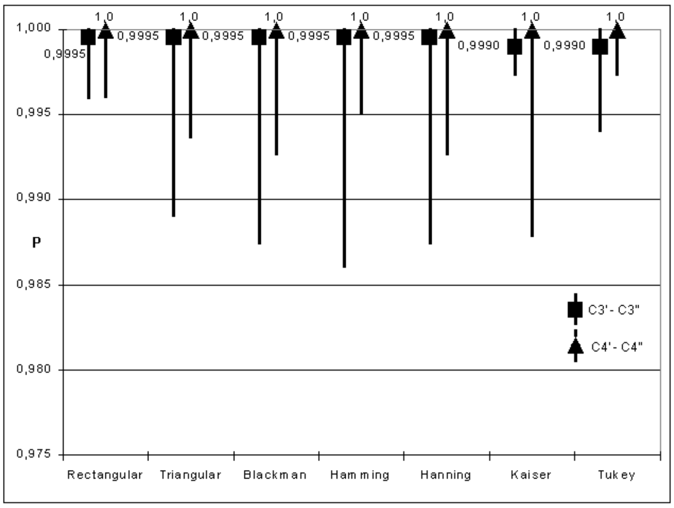

Fig. 4. Math task vs. Relax. Coordinate X1.

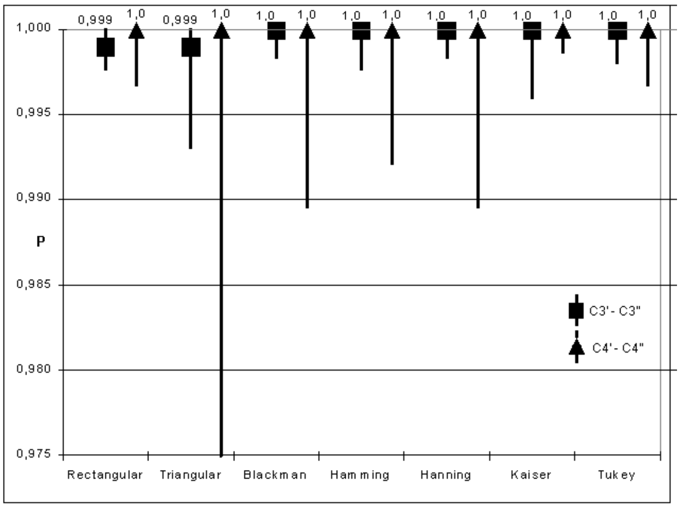

Fig. 5. Motor imagery vs. Relax. Coordinate X1.

VI. Discussion.

In all cases only two eigen-values have got significant magnitudes, so only two eigen-vectors have been considered in the transformation matrix. This causes that LDA technique had projected the original six dimensional feature space over a bidimensional space, maintaining the intrinsic characteristics of each cerebral activity.

The analysis of the results for the transformed coordinate X_1, shows that with a significance level of $\alpha = 2.5\%$, $\alpha = 1 - p$, in almost all cases the null hypothesis H_o, which maintains the equality in the populations of the features associated to mental tasks, should be rejected. On the other hand, the same analysis for X_2 shows that the difference appears only in some cases. It is also shown that channel C4'-C4" performs better than C3'-C3".

On average all filtering windows show statistical difference between mental tasks; the best results, with higher quantities for the mode values and lower dispersion, are obtained for X_1 with Tukey's and Kaiser's filtering windows, the worst results are obtained with Blackman's, Hamming's and Hanning's windows. It is observed that as higher the eigen-value magnitude, case of X_1, the higher the value of one component of the eigen-vector, normally in β frequency band, by the contrary, as lower the eigen-value more the contribution of the rest of eigen-vector components.

The highest contrast power is obtained in the comparison of *Motor imagery* vs. *Relax*, it is followed by *Mathematical task* vs. *Relax*, and the lowest is for *Mathematical task* vs. *Motor imagery*.

VII. Conclusions.

This report shows a methodology, based on LDA technique, which weight the power amplitude of frequency bands, and at the same time, allow to reduce the feature input space maintaining the particularities of the considered cerebral activities. On the other hand, eigen-vector analysis shows that the discrimination

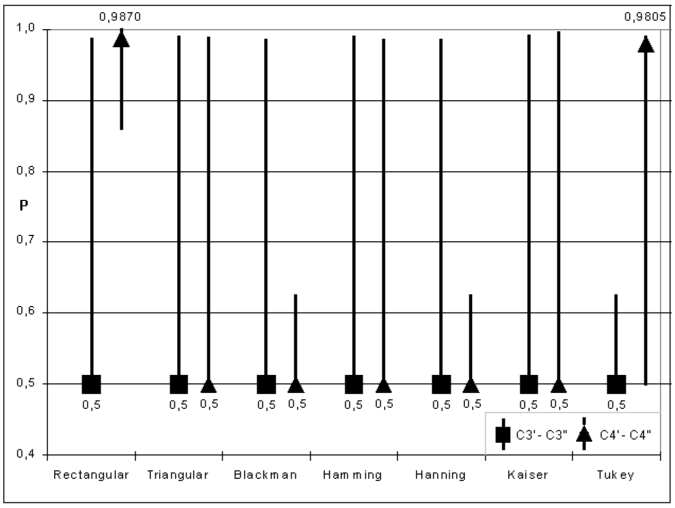

Fig. 6. MATH TASK VS. MOTOR IMAGERY. COORDINATE X2.

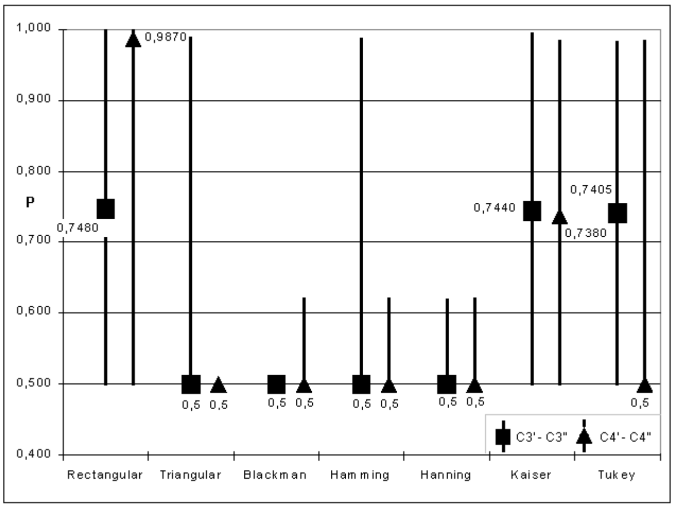

Fig. 8. MOTOR IMAGERY VS. RELAX. COORDINATE X2

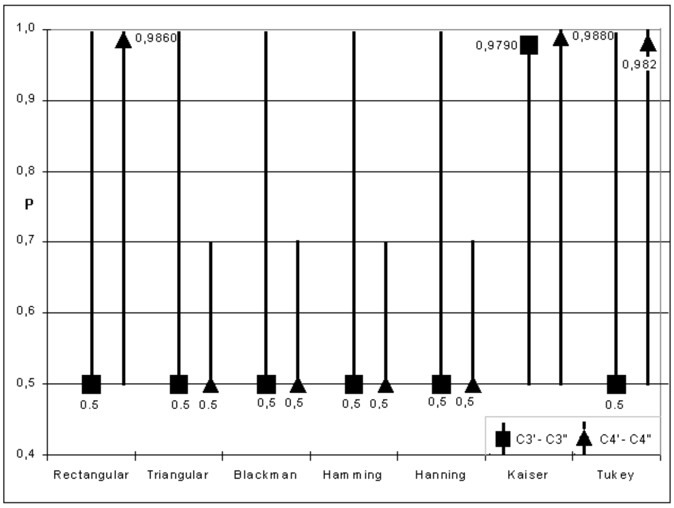

Fig. 7. MATH TASK VS. RELAX. COORDINATE X2.

power is manifested over β band, mainly β_2 and β_3, they are the vector components with highest contributions in the transformation matrices.

In this report, significance levels from bilateral contrast tests are used as quantifiers for the discrimination capability, they corroborate the conclusions of [14] taking in consideration more subjects and experiments; they establish that registers of encephalographic signal, meanwhile the subjects are thinking on different mental activities, are different and it is possible to differentiate between them; this is a key evidence for the developing process of On-line BCI devices; they also show that Tukey's and rectangular filtering windows improve the discrimination capability between the considered mental tasks.

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